Section 6.2
Antidifferentiation by Substitution

Definite Integral vs. Indefinite Integral

Definite Integral is a number, the limit of a sequence of Riemann Sums

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Indefinite Integral is a family of functions having a common derivative.

$$\int f(x)\,dx = F(x) + C$$

Example

$$\int_0^0 x^2 - \sin x \, dx$$

$$= \frac{x^3}{3} + \cos x + C$$

General Antiderivative - Indefinite Integral

$$\int x^2\,dx = \frac{x^3}{3} + C$$

$$\int \frac{1}{x+1}\,dx = \ln|x+1| + C$$

$$\int \frac{1}{x^3}\,dx = -\frac{1}{2x^2} + C$$

$$\int \cos(2x)\, dx = \frac{\sin(2x)}{2} + C$$

$$\int e^{3x}\, dx = \frac{e^{3x}}{3} + C$$

$$\int x^2 + 7\, dx = \frac{x^3}{3} + 7x + C$$

$$\int \sin(2x)\,dx = -\frac{\cos(2x)}{2} + c$$

$$\int e^{(1/3)x}\,dx = 3e^{\frac{1}{3}x} + c$$

Properties of Indefinite Integrals

$$\int k f(x) dx = k \int f(x)\, dx \quad \text{for any constant } k$$

$$\int (f(x) \pm g(x))\, dx = \int f(x)\, dx \pm \int g(x)\, dx$$

Power Formulas

$$\int u^n \, du = \frac{u^{n+1}}{n+1} + C \text{ when } n \neq -1$$

$$\int u^{-1} \, du = \int \frac{1}{u} \, du = \ln |u| + C$$

$$\int x^2 \, dx$$

$$\frac{x^{2+1}}{2+1} + C$$

$$\frac{x^3}{3} + C$$

Trigonometric Formulas

$$\int \cos u \, du = \sin u + C \qquad \int \sin u \, du = -\cos u + C$$

$$\int \sec^2 u \, du = \tan u + C \qquad \int \csc^2 u \, du = -\cot u + C$$

$$\int \sec u \tan u \, du = \sec u + C \qquad \int \csc u \cot u \, du = -\csc u + C$$

Exponential and Logarithmic Formulas

$$\int e^u\,du = e^u + C$$

$$\int a^u\,du = \frac{a^u}{\ln a} + C$$

$$\int \ln u\,du = u\ln u - u + C \quad \text{(See Example 2)}$$

$$\int \log_a u\,du = \int \frac{\ln u}{\ln a}\,du = \frac{u\ln u - u}{\ln a} + C$$

$$x\ln x - x + c$$

$$= (1)\ln x + x\cdot\frac{1}{x} - 1$$

$$= \ln x + 1 - 1$$

$$= \ln x$$

More Examples

$$\int u^{-1}\, du =$$

$$\int \ln u\, du =$$

Let $f(x) = x^3 + 1$ and let $u = x^2$. Find each of the following antiderivatives in terms of x:

a.) $\int f(x)\,dx = \int (x^3+1)\,dx = \frac{1}{4}x^4 + x + C$

b.) $\int f(u)\,du = \int (u^3+1)\,du = \frac{u^4}{4} + u + C$

$= \frac{(x^2)^4}{4} + x^2 + C$

$= \frac{x^8}{4} + x^2 + C$

c.) $\int f(u)\,dx =$

$$\int \cos(x^3)\, dx =$$

Substitution in Indefinite Integrals

$$\int \cos(x^3) * x^2\,dx =$$

Let $u = x^3$

$$\frac{du}{dx} = 3x^2$$

$$du = 3x^2\,dx$$

$$\frac{du}{3} = x^2\,dx$$

$$\int \cos u \cdot \frac{du}{3}$$

$$= \frac{1}{3}\int \cos u\,du = \frac{1}{3}\sin u + C$$

$$= \frac{1}{3}\sin x^3 + C$$

$$\int \tan x \, dx = \int \frac{\sin x}{\cos x} dx = \int \frac{1}{\cos x} \cdot \sin x \, dx$$

$$u = \cos x \qquad \frac{du}{dx} = -\sin x$$

$$du = -\sin x \, dx$$

$$-du = \sin x \, dx$$

$$= \int \frac{1}{u} (-du) = -\int \frac{1}{u} du = -\ln|u| + C$$

$$= -\ln|\cos x| + C$$

$$\int 2x\sqrt{4 - x^2}\, dx =$$

$u = 4 - x^2$

$\frac{du}{dx} = -2x$

$du = -2x\,dx$

$-du = 2x\,dx$

$-\int \sqrt{u}\, du$

$= -\int u^{\frac{1}{2}} du = -\frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C$

$= \frac{-2u^{\frac{3}{2}}}{3} + C = \frac{-2(4-x^2)^{\frac{3}{2}}}{3} + C$

$$\int \frac{4e^x}{\sqrt{e^x}}\,dx = 4\int \frac{e^x}{\sqrt{e^x}}\,dx$$

$$u = e^x \qquad \frac{du}{dx} = e^x$$

$$du = e^x\,dx$$

$$4\int \frac{1}{\sqrt{u}}\,du = 4\int u^{-\frac{1}{2}}\,du$$

$$= 4\,\frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C = 8u^{\frac{1}{2}} + C$$

$$= 8\sqrt{e^x} + C$$

$$\int \sin x \, e^{\cos x} \, dx =$$

$u = \cos x$ $\qquad \frac{du}{dx} = -\sin x$

$du = -\sin x \, dx$

$-du = \sin x \, dx$

$$\int e^u \cdot (-du) = -e^u + C$$

$$= -e^{\cos x} + c$$

$$\int x^2 \sqrt{5 + 2x^3}\, dx =$$

$u = 5 + 2x^3$

$$\frac{du}{dx} = 6x^2$$

$$du = 6x^2\, dx$$

$$\frac{du}{6} = x^2\, dx$$

$$\int \sqrt{u}\, \frac{du}{6} = \frac{1}{6} \int u^{\frac{1}{2}}\, du = \frac{1}{6} \cdot \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$$= \frac{1}{9} (5 + 2x^3)^{\frac{3}{2}} + c$$

$$\int \cot 7x \, dx =$$

Find the indefinite integrals. In each case you can use a trigonometric identity to set up a substitution.

$$\int \frac{dx}{\cos^2 2x} =$$

$$\int \cot^2 3x \, dx =$$

$$\int \cos^3 x \, dx =$$

Substitution in Definite Integrals

Evaluate $$\int_0^{\pi/3} \tan x \sec^2 x \, dx =$$

$$\int_0^{\pi} e^{\sin x} \cos x \, dx =$$

Evaluate $$\int \frac{t}{\sqrt{2t^2 + 1}}\, dt =$$

$$\int \frac{\ln x}{x}\, dx =$$

$$\int \frac{\sin(2t+1)}{\cos^2(2t+1)}\,dt =$$

$$\int_{2}^{5} \frac{\sqrt{\ln x}}{x}\, dx =$$

Evaluate $$\int_0^1 \frac{x}{x^2 - 4}\, dx =$$